Lecture 2 - Properties and representation of a game

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Introduction

- When players design their strategies they have to take the other players' into account
- They establish anticipations which - more or less - match with really what happens at the end
- Anticipations here mainly depends on two features
  - Information
  - Rationality
- We next define these two features and provide taxonomies

1. Rationality

- Individual chooses his action consistently
- he acts as if he maximized his utility function considering both his befiefs and his environment.
- he chooses the best action according to his preferences, among all the actions available to him

1.1 Preferences

- we assume that a player, when presenting with any pair of actions, knows which of the pair he prefers, or knows that he regards both actions as equally desirable
- transitivity principle.
- No other restriction is imposed on preferences.

1.1 Preferences

- preferences are represented by a payoff function, which associates a number with each action in such a way that actions with the highest number are prefered
- Player's preferences in the sense used here convey only **"ordinal information"**

1.2 Instrumental rationality

- refers to the ability of a player to calculate strategies from his beliefs.

1.3 Cognitive rationality

- refers to the ability of a player to perceive the situation of the game in which she has to take a decision

2. Information

The main charateristic of the environment is the nature of information. We distinguish :

- Perfect from imperfect information
- Complete from incomplete information

Perfect vs. imperfect information

- We refer to the rules of the game.
- Perfect information means that players knows exactly what did happen in the past when they make their decision.

2. Information

Complete vs. incomplete information

- We refer to the circumstances of the game.
- Under complete information, the structure of the game and the payoff functions of the players are commonly known but players may not see all of the moves made by other players.

2. Information

- Games of incomplete information can be converted into games of complete but imperfect information under the common prior assumption.
- In the transformed game, all players know the new meta-rules, including the fact that Nature has made an initial move unobserved by them.

- studying dynamic games in which some decisions are made after others
- allows us to make explicit this temporal structure as well as to define the game's information structure
- Extensive form games model multi-agent sequential decision making
- game tree is the supporting framework for the extensive form games.

Strategic form games are useful when decision order is irrelevant:

- Simultaneous move
- Sequential move but impossible to condition on previous behaviour
- Repeated without interaction between stages

Such games are also referred to as normal form games or matrix games.

Assuming

- 2 players : 1 and 2
- Each player has 2 actions: Player 1 can choose between A and B and player 2 between C and D

Table: Strategic form game

		Player 2	
		C	D
Player 1	A	$u_1(A;C);u_2(A;C)$	$u_1(A;D);u_2(A;D)$
	B	$u_1(B;C);u_2(B;C)$	$u_1(B;D);u_2(B;D)$

3.2.3. A particular case: Symmetric games

Definition: A two-player strategic game with ordinal preferences is symmetric if the players' sets of strategies are the same and the players' preferences are represented by payoff functions u_1 and u_1 for which $u_1(s_1, s_2) = u_2(s_2, s_1)$ for every strategy pair (s_1, s_2).

3.2.4. Example - The prisoner dilemma game

Two suspects in a major crime are help is separate cells. There is enough evidence to convict each of them of a minor offense but not enough evidence to convict either of them of the major crime unless one of them acts as an informer against the other (finks). If both stay quiet, each will be convicted of the minor offense and spend 1 year in prison. If one and only one of them finks, he will be freed and used as a witness against the other, who will spend 4 years in prison. If they both fink, each will spend three years in prison.

3.2.4. Example - The prisoner dilemma game

Such situation can be model as a strategic game.

- Players: 2 suspects
- Strategies (or actions): Each player's set of actions is quiet; Fink
- Preferences: Suspect 1's ordering of the action profiles, from best to worst, is:
  - (Fink, Quiet): he finks and suspect 2 remains quiet so he is freed
  - (Quiet; Quiet): he gest one year in prison
  - (Fink; Fink): he gets 3 years in prison
  - (Quiet, Fink): he gets 4 years in prison
  - Suspect 2's ordering is : (Quiet, Fink), (Quiet; Quiet), (Fink; Fink), (Fink, Quiet)

3.2.4. Example - The prisoner dilemma game

It results the following strategic form game:

Table: Strategic form of the prisoner dilemma game

		Suspect 2	
		Quiet	Fink
Suspect 1	Quiet	2,2	0,3
	Fink	3,0	1,1

3.2.5.From the extensive form game to the strategic one

- can always associate to an extensive form game a strategic form.
- In a two-player game, to a strategic form game, we can associate 2 extensive form depending on the order of moves of players.
- Particular case: simultaneous moves games.

3.2.6. The 3-player case

Assuming now 3 players, having each 2 strategies

- Player 1 has strategies s_1 and s_1'
- Player 2 has strategies s_2 and s_2'
- Player 3 has strategies s_3 and s_3'

Leading to the following payoffs functions

- Player 1 : $u_1(s_1, s_2, s_3)$, $u_1(s_1', s_2, s_3)$, $u_1(s_1', s_2', s_3)$....
- Player 2 : $u_2(s_1, s_2, s_3)$, $u_2(s_1', s_2, s_3)$, $u_2(s_1', s_2', s_3)$....
- Player 3 : $u_3(s_1, s_2, s_3)$, $u_3(s_1', s_2, s_3)$, $u_3(s_1', s_2', s_3)$....

3.2.6. The 3-player case

The corresponding strategic form is as follows:

Figure: Strategic form with a 3-player game

	s_2	s'_2
s_1	$u(s_1, s_2, s_3)$	$u(s_1, s'_2, s_3)$
s'_1	$u(s'_1, s_2, s_3)$	$u(s'_1, s'_2, s_3)$

s_3

	s_2	s'_2
s_1	$u(s_1, s_2, s'_3)$	$u(s_1, s'_2, s'_3)$
s'_1	$u(s'_1, s_2, s'_3)$	$u(s'_1, s'_2, s'_3)$

s'_3

$$u(\cdot) = (u_1(\cdot),\ u_2(\cdot),\ u_3(\cdot))$$

4. Conclusion

- In strategic interactions, players are supposed to act in a rational way
- Players express preferences that provide only ordinal information over the ordering of choices.
- Strategic interactions can be modelized with extensive form game in case of sequential order of moves
- Both simultaneous and sequential order of moves can be modelized trhough a strategic form game.