Tutorial 3 – Due to November 10th, 2016

Please respond in detail to each question. Any unjustified or unexplained response will be considered as false.

Exercise 1

Assume a Cournot duopoly in which 2 firms are on the market and produce an homogeneous product.

- The strategy of firm 1 is the quantity of product it will produce: $q_1 \in [0; \infty]$
- The strategy of firm 2 is the quantity of product it will produce: $q_2 \in [0; \infty]$
- The profit of each firm is equal to his revenue minus his total cost
- The inverse demand function of the market is :$P(Q) = -q_1 - q_2 + 100$ with $Q = q_1 + q_2$
- The total cost function is the same for the two firms and equal to: $CT(q_1) = -4q_1 + q_1^2$ and $CT(q_2) = -4q_2 + q_2^2$

By assuming that the two firms choose the level of production simultaneously determine the quantity produced by each firm, the price equilibrium and the profit of each firm. You will represent on a figure this equilibrium.

Exercise 2

2 firms decide to launch a new product. Each firms has to choose a strategy between : low price, high design and high quality, denoted respectively by P, D and Q. According to market studies their profits will be as follows:

Table 1:

		Firm 2		
		P	D	Q
Firm 1	P	0,0	1,-1	-1,1
	D	-1,1	0,0	1,-1
	Q	1,-1	-1,1	0,0

1. Check there is no Nash equilibrium in pure strategy
2. Determine the Nash equilibrium in mixed strategy

Exercise 3

There is a shared meadow with a maximum capacity. To simplify, we set the maximum capacity equal to 1. Each player may wish to put f_i cows in the meadow. The maximum capcity of 1 does not mean that it is possible only to put 1 cow in the meadow. We have:

$$\begin{aligned} f_i &\in [0,1] \qquad (1) \\ N &= \{1,2,...,n\} \\ S1 = S2 = ..., = Sn &= [0,1] \end{aligned}$$

- If $\sum_{i \in N} f_i > 1$ the maximum capacity is exceeded and the payoff of each player is equal to 0
- If $\sum_{i \in N} f_i < 1$, we consider the following payoff

$$u_i = f_i(1 - \sum_j f_j)$$

From the payoff of player i, we note that the payoff of player i is proportional to the number of cows he puts (f_i) but it is also inversely proportional to the total number of cows ($\sum_j f_j$). Note that the total number of cows, $\sum_j f_j$, includes f_i.

1. Determine the Nash equilibrium (f^*).
   - Consider only the case where the maximum capacity is not exceeded
   - To get the first order condition, you can put $t = \sum_{j \neq i} f_j$
2. Determine the payoff at the Nash equilibrium for each player
3. Determine the total payoff (the sum of payoff for the n players) at the Nash equilibrium
4. Compare this total payoff with the total payoff resulting from the strategy profile $f = \left\{\frac{1}{2n}, \frac{1}{2n}, \cdots, \frac{1}{2n}\right\}$
5. Comment your results